New Generalizations of Besov-type spaces and Triebel-Lizorkin-type spaces

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Let $0 < p < \infty$, $0 < q \leq \infty$ and $s \in \mathbb{R}$. We introduce a new type of generalized Besov-Triebel-Lizorkin-type spaces $A_{p,q}^{s,\varphi}(\mathbb{R}^n)$, where φ belongs to the class $\mathcal{G}_p$, that is, $\varphi : (0,\infty) \rightarrow (0,\infty)$ is increasing and $t^{-n/p}\varphi(t)$ is decreasing in $t > 0$.

We start from the well-known Besov-Triebel-Lizorkin-type spaces $A_{p,q}^{s,\tau}(\mathbb{R}^n)$, $\tau > 0$ and replace $|Q|^\tau$ in their definition by $\varphi(\ell(Q))$, where Q is some dyadic cube with volume $|Q|$ and side length $\ell(Q)$. We establish several basic properties of the spaces $A_{p,q}^{s,\varphi}(\mathbb{R}^n)$ and investigate the relations within that scale of spaces, as well as to some classical function spaces, especially (generalized) Besov-Triebel-Lizorkin-Morrey spaces.